

# Computation of Reidemeister classes by nilpotentization

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## Abstract

The Reidemeister trace is an algebraic invariant of a self-map that provides both the Nielsen and Lefschetz numbers. Fadell and Husseini showed that the Fox calculus can be used for a partial computation of the Reidemeister trace on surfaces. The remaining step in the computation consists of a “twisted conjugacy” problem: given words  $x$  and  $y$  in a free or one-relator surface group and a homomorphism  $\varphi$ , decide whether or not there is some word  $z$  with

$$\phi(z)xz^{-1} = y.$$

A standard technique for computation of the twisted conjugacy classes (called “Reidemeister classes”) is abelianization, which unfortunately disregards much of the structure of the underlying group. We explore an improvement of the abelianization technique, which we call “nilpotentization.” Instead of a projection into the abelianization, we make use of classical techniques of P. Hall to perform computations in nilpotent quotients.

The nilpotentization technique leads to a method both for distinguishing and for equating Reidemeister classes that is generally applicable, and much more successful than abelianization. We also present results of a computational implementation of the technique in the MAGMA computer algebra system.