Elegant ideas, and why you should love them

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Introduction

Mathematical beauty is not just for mathematicians

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We'll talk about:

- Some elegant ideas ("Green noses")
- Similar types of beauty in other arts

An elegant sandwich

My lunch box.

It should fit, but it's the wrong shape.

How to make it fit?

How to make it fit?

Here's one method:

How to make it fit?

Here's one method:

Not elegant.

A better solution:

A better solution:

A better solution:

A better solution:

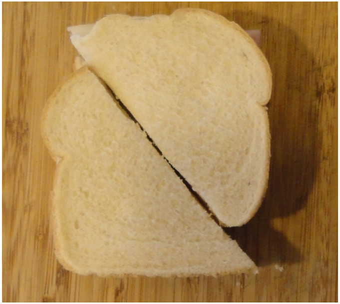
 

Better.

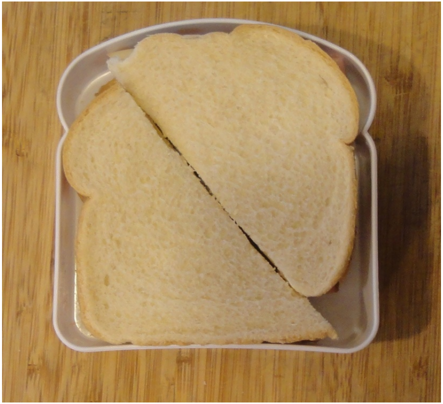
An elegant solution:

An elegant solution:

An elegant solution:



An elegant solution:



(hold your applause)

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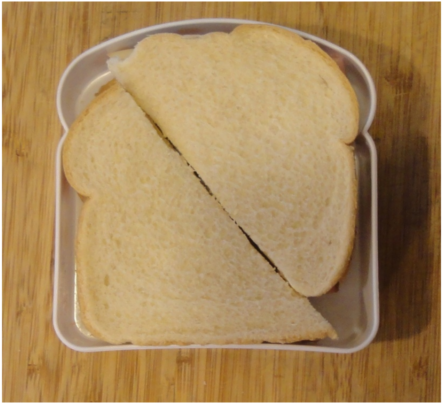

(hold your applause)

“Elegance” is simple, effortless beauty.

If you can be even slightly impressed by this example, mathematical beauty is for you.

Green noses

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"Give me the child when he is seven, and I will give you the man." – Ignatius of Loyola

"Still looking for the green noses."

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Most people disappointed by mathematics.

"Still looking for the green noses."

Most people disappointed by mathematics.

But there are lots of green noses.

"Still looking for the green noses."

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Here's three examples:

Green nose #1: An elegant shape

Here's a picture you may have seen before:

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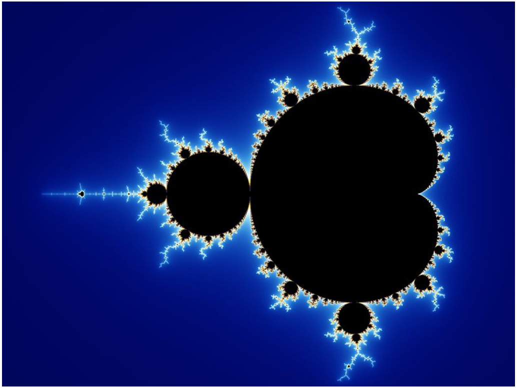

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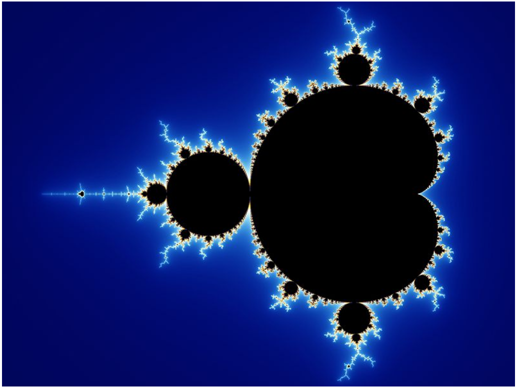

The Mandlebrot set. (It's a fractal.)

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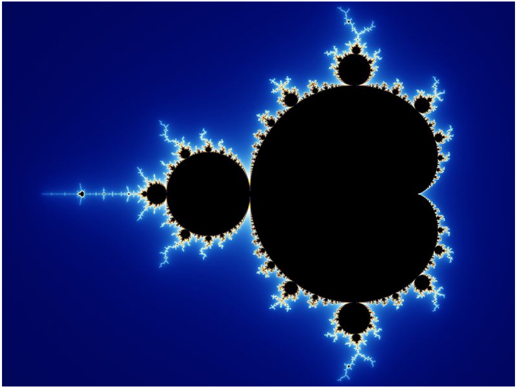

The Mandlebrot set. (It's a fractal.)

"The most complicated object in mathematics" *Scientific American, 1986*

So what?

So what? I can do that.

So what? I can do that.

So what? I can do that.

The Staeckerbrot!

So what? I can do that.

The Staeckerbrot! Not really a "mathematical" shape.

The Mandlebrot set can be described by an equation.

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And the equation is very very simple:

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$$f(z) = z^2 + c$$

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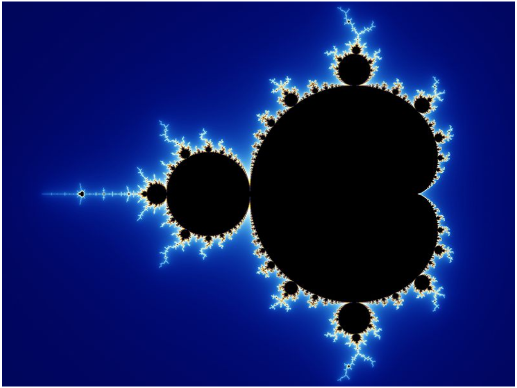

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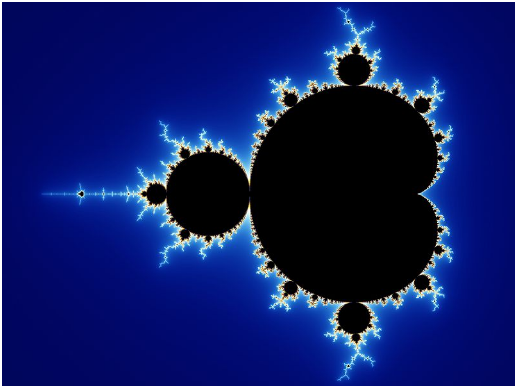

Boneheadedly simple, but shockingly complex.

Why does such complexity exist in mathematics?

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If we created mathematics, then why does it surprise us?

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These surprises are beautiful.

Green nose #2: An elegant way of thinking

Let's try a little arithmetic:

$$\begin{array}{r} 344 \\ +\quad 217 \\ \hline \end{array}$$

Green nose #2: An elegant way of thinking

Let's try a little arithmetic:

$$\begin{array}{rr} & {\scriptstyle 1} \\ & 344 \\ + & 217 \\ \hline & 1 \end{array}$$

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Let's try a little arithmetic:

$$\begin{array}{rr} & {\scriptstyle 1} \\ & 344 \\ + & 217 \\ \hline & 61 \end{array}$$

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Let's try a little arithmetic:

$$\begin{array}{rr} & {}^{1} \\ & 344 \\ + & 217 \\ \hline & 561 \end{array}$$

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You could probably even do this in your head, the same way.

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Awesome, right?

Imagine you're a Roman centurion, and you want to add these numbers:

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Try:

$$\begin{array}{rr} & \mathtt{CCCXXXIV} \\ + & \mathtt{CCXVII} \\ \hline \end{array}$$

How about this:

$$2355 \div 3$$

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Here goes:

$$3\ \overline{)2355}$$

How about this:

$$2355 \div 3$$

Here goes:

$$3\ \overline{)2355}^{\;\;7}$$

How about this:

$$2355 \div 3$$

Here goes:

$$\begin{array}{r} 7 \\ 3\,\overline{)\,2355} \\ \underline{21} \end{array}$$

How about this:

$$2355 \div 3$$

Here goes:

$$\begin{array}{r} 7 \\ 3\ \overline{)2355} \\ \underline{21} \\ 2 \end{array}$$

How about this:

$$2355 \div 3$$

Here goes:

$$\begin{array}{r} 7 \\ 3\,\overline{)2355} \\ \underline{21} \\ 25 \end{array}$$

How about this:

$$2355 \div 3$$

Here goes:

$$\begin{array}{r} 78 \\ 3 \;\big)\overline{2355} \\ \underline{21} \\ 25 \end{array}$$

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$$\begin{array}{r} 78 \\ 3\ \overline{)2355} \\ \underline{21} \\ 25 \\ \underline{24} \end{array}$$

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$$\begin{array}{r} 78 \\ 3\;\overline{)2355} \\ \underline{21} \\ 25 \\ \underline{24} \\ 1 \end{array}$$

How about this:

$$2355 \div 3$$

Here goes:

$$\begin{array}{r} 78 \\ 3\ \overline{)2355} \\ \underline{21} \\ 25 \\ \underline{24} \\ 15 \end{array}$$

How about this:

$$2355 \div 3$$

Here goes:

$$\begin{array}{rr}
 & 785 \\ \cline{2-2}
3 & \big)2355 \\
 & \underline{21} \\
 & 25 \\
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 & 15
\end{array}$$

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$$
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 & 25 \\
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\end{array}
$$

Ask the centurion:

$$\texttt{III}\overline{)\texttt{MMCCCLV}}$$

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The centurion probably will be unable to do this without mechanical help.

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Your advantage is having an elegant way of thinking.

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Roman numerals are a bit like counting change: everything in groups of 1, 5, 10, etc.

What's 3 quarters, 4 dimes, 1 nickel and 2 pennies plus 2 quarters, 3 dimes, and 4 pennies?

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'The best design is an invisible one'

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"The Compendious Book on Calculation by Completion and Balancing"

Book includes the quadratic formula, and lots of applications to geometry and Islamic inheritance.

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It took a long time for the arabic numerals to become popular in Europe.

Gregor Reisch, *Typus Arithmeticae*, 1525

Fibbonacci (13th century) helped spread them to Europe.

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Elegant ways of thinking are not always recognized, even by very smart people.

Green nose #3: An elegant formula

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Bernoulli was working on compound interest.

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$$i = \sqrt{-1}$$

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So we have π from geometry, e from analysis, and i from algebra.

Each concept was developed by different people to solve completely different problems. There should be no relationship between them.

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It is beautiful.

From whence the green noses?

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Reminds me of Scooby Doo.

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Maybe there is something else at work here...

Maybe mathematics is just part of the universe.

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A deep part.

Maybe mathematics is just part of the universe.

A deep part.

More fundamental than the laws of physics.

Maybe mathematics is just part of the universe.

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More fundamental than the laws of physics.

But why should it be beautiful?

Maybe God made it that way.

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This could explain why it's beautiful:

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God values beauty, and so God made the creation beautiful at the most fundamental level possible.

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But "because God did it" or "because God said so" shouldn't be satisfying answers.

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In other contexts, these answers are dangerous.

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With or without God in the picture, the existence of mathematical beauty is a fundamental mystery that should inspire us and humble us.

Mathematical beauty and other types of beauty

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First: what exactly is mathematics about?

What is it?

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About learning how to think appropriately.

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Certainly numbers are mathematical, but they are only a part of mathematics in general.

Mathematics generally is about patterns and structured reasoning.

About learning how to think appropriately.

Certainly numbers display patterns and require structured reasoning, but this is only one setting.

Deeper magic

Mathematics is a deeper magic.

Deeper magic

Mathematics is a deeper magic.

The Lion, The Witch, and the Wardrobe, C.S. Lewis

Beauty

The basic themes of structure and patterns are universal.

Beauty

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Let's look at some other beautiful arts which are beautiful in similar ways to mathematics.

Structured beauty

Mathematical research is creative, but strongly *structured*.

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All mathematics must lie within the rules of logical reasoning.

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"Physics is imagination in a straightjacket" – Moffat (1939–)

Is true artistry possible within strict structures?

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Poetry

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Would Shakespeare's works have been better if he hadn't written in meter?

Is true artistry possible within strict structures?

Poetry

Would Shakespeare's works have been better if he hadn't written in meter?

In the hands of the artist, the structure becomes a strength rather than a weakness.

Often, restrictions often make the art better.

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Star Wars episode IV (1977) vs.

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Films of Lars von Trier, *The Five Obstructions*

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Cinema Verité, etc.

To a lesser extent, any visual art which incorporates its surroundings is like this.

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Cave art flows with the contours of the walls.

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Cave art flows with the contours of the walls.

Architecture and graffiti art use the existing landscape.

Deterministic beauty

Art in the landscape:

Getty Center Museum, Los Angeles. (photo: http://academic.reed.edu/getty/)

Deterministic beauty

Art in the landscape:

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Deterministic beauty

Art in the landscape:

Israeli West Bank barrier. (photo: Wikipedia)

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Like mural-making.

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The most beautiful facts will touch the surrounding landscape in new and unexpected ways. (Euler's identity)

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Like Michelangelo: the sculpture already exists inside the block, we just need to "free the idea" by chipping away.

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Determinism is sometimes used in music:

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Ligeti, *Pome Symphonique for 100 metronomes*, 1962.

Can something fundamentally deterministic be truly beautiful?

Determinism is sometimes used in music:

Ligeti, *Pome Symphonique for 100 metronomes*, 1962. (Not conventionally beautiful to listen to.)

Can something fundamentally deterministic be truly beautiful?

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Ligeti, *Pome Symphonique for 100 metronomes*, 1962. (Not conventionally beautiful to listen to.)

Steve Reich, *It's Gonna Rain* (1965), *Come Out* (1966)

Pärt, *Cantus in Memoriam Benjamin Britten*, 1977.

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First violin part:

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First violin part:

Pärt, *Cantus in Memoriam Benjamin Britten*, 1977.

First violin part:

a few pages later...

Pärt, *Cantus in Memoriam Benjamin Britten*, 1977.
Second violin part:

Pärt, *Cantus in Memoriam Benjamin Britten*, 1977.
Second violin part:

Same pattern, half speed

Pärt, *Cantus in Memoriam Benjamin Britten*, 1977.
Viola part:

Same pattern, one-fourth speed

The cello plays the same pattern at one-eighth speed,

The cello plays the same pattern at one-eighth speed, the bass at one-sixteenth speed.

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But it sounds beautiful, and not at all artificial.

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This is a beautiful mystery.

That's all!

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