Voting: How it works, and why it doesn't

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Fairfield U. Mathematics & Computer Science Colloquium
Election Day 2012

This talk is about fairness in voting systems.

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I'll discuss specifically the unfairness in our system of voting.

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I also will not discuss the electoral college.

This is a crazy overlay onto our basic voting system which makes everything slightly weirder.

I'm interested in the system at a much more fundamental level.

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Actually, voting is an insane idea when you think about it.

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This sounds sketchy.

Something that complicates everything:

Preferences of groups of people do not behave like preferences of individual people.

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This is the *Condorcet paradox*.
(Condorcet, 1743-1794)

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Imagine an election with three candidates A, B, C.

No person would ever say: "I like A more than B, and B more than C, and C more than A".

Individual preferences are *transitive*.

But let's ask a group of people to rank their choices, and imagine they say:

15	11	13
A	B	C
B	C	A
C	A	B

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So what is the "will of the people"?

Sounds like there is no coherent will of the people.

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- If the society actually has a uniform preference, the decision should reflect this.

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Basically, a winner-selection method should analyse the preferences, and choose a winner based on some relevant details of the set of preferences.

For a reasonably fair system:

- If the society actually has a uniform preference, the decision should reflect this.
- The decision should not depend on irrelevant details of the preferences.

Let's vote!

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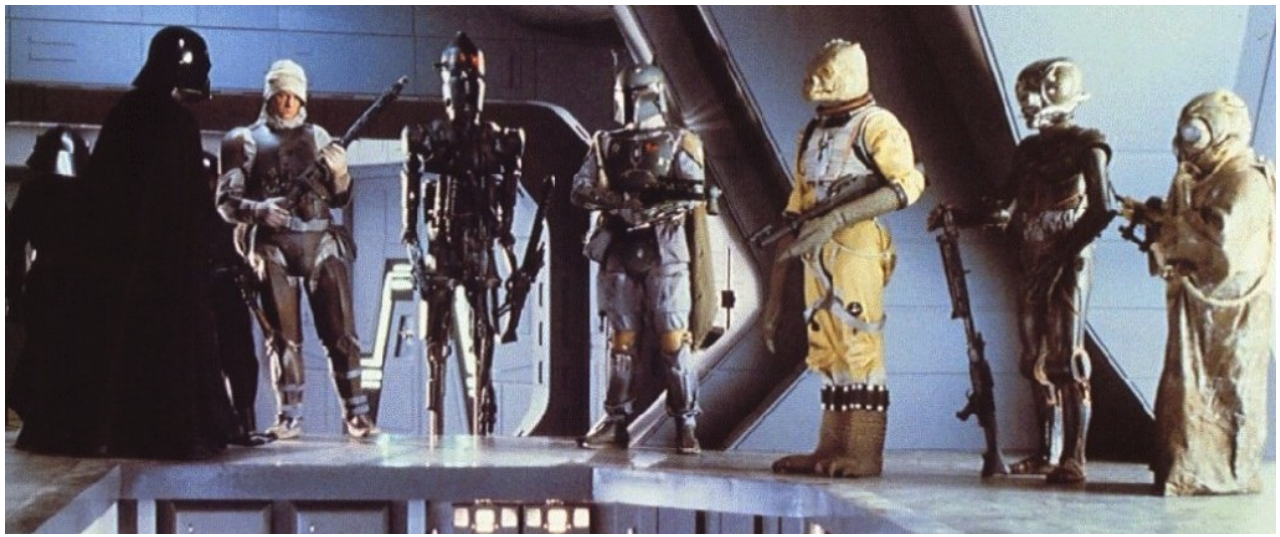
The second-best bounty hunter from *The Empire Strikes Back*



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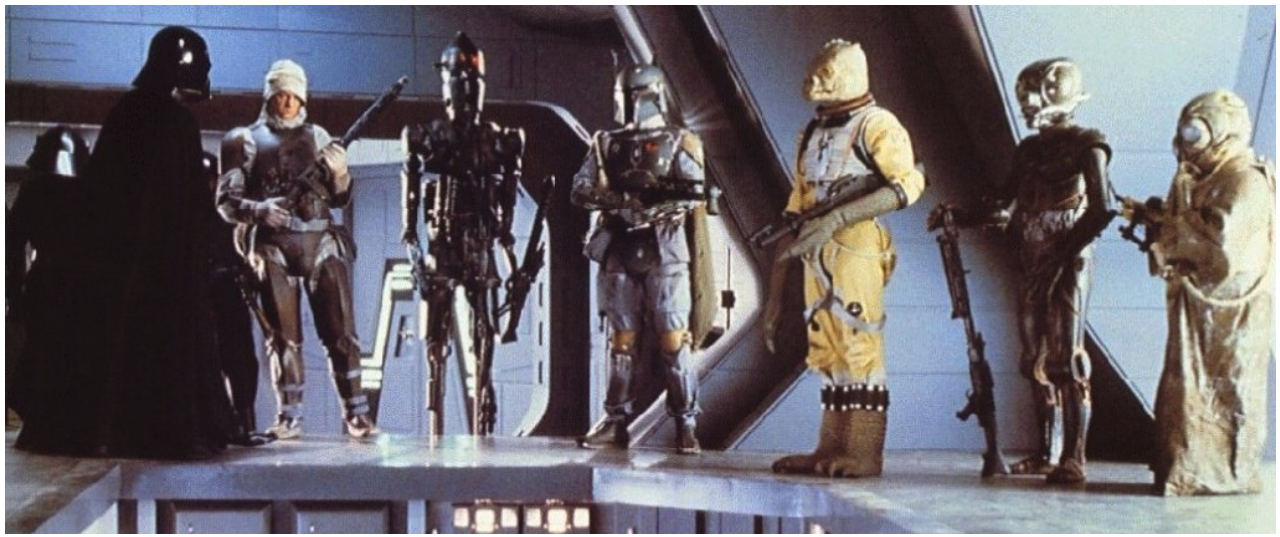

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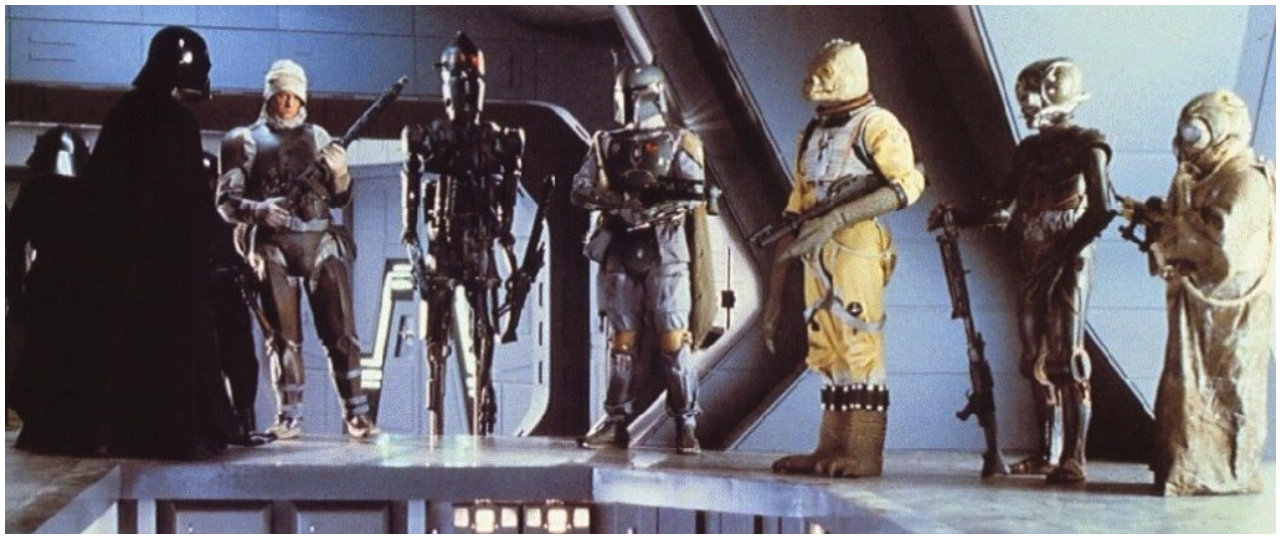

Obviously Boba Fett is the best. We'll vote for second best.

Here are the choices:

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Bossk

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Zuckuss

Here are the choices:

Bossk

4-LOM

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Here are the choices:

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IG-88

To make it interesting, let's *rank* our choices.

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Choose your #1, #2, etc. choice.

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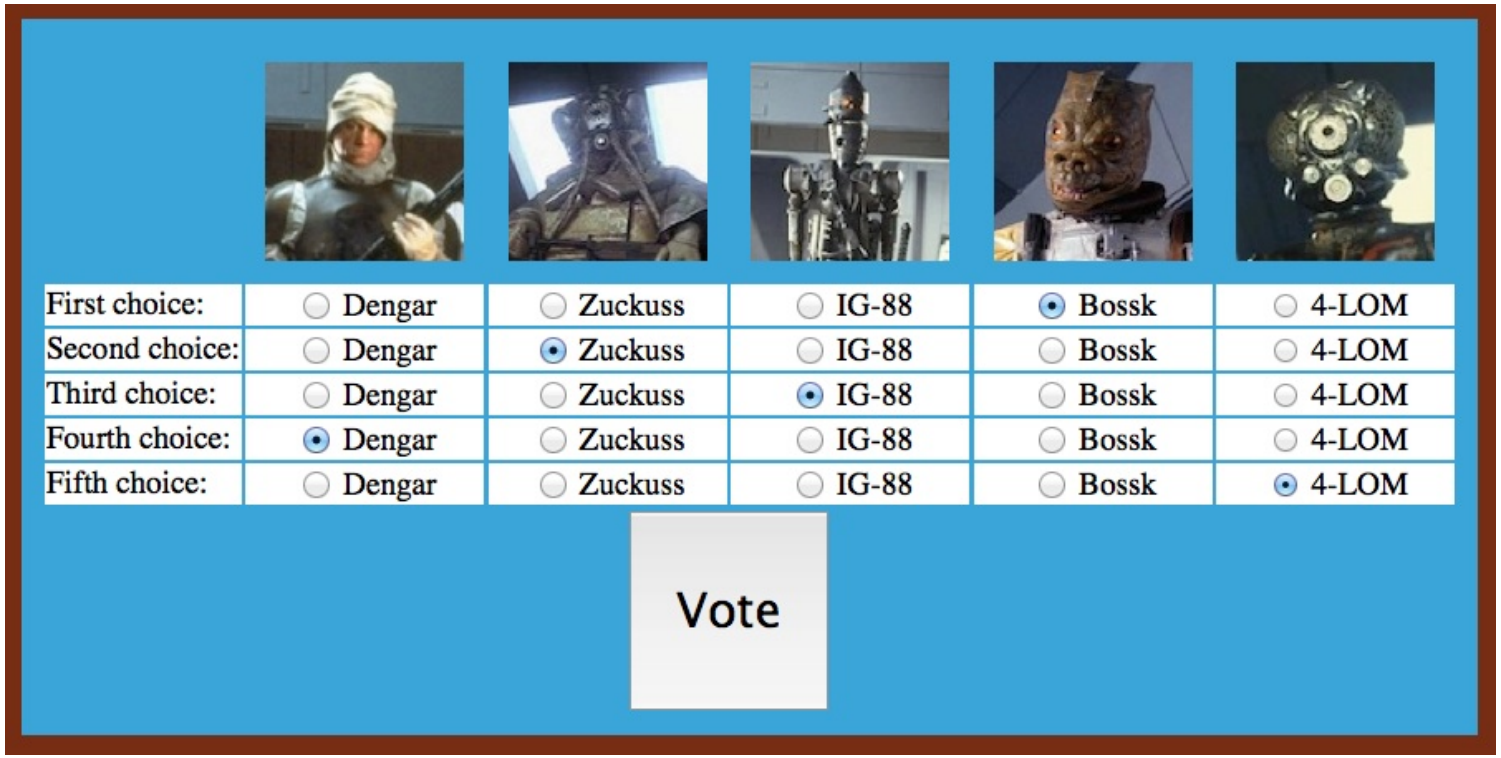
After we vote, we'll count up the votes and have our decision.

Your ballot will look like this:



Vote on the tablets going around, or:

Connect to the "staecker" wi-fi network, and visit:
`http://staecker.local/vote`

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Even reasonable alternative systems will produce wildly different outcomes.

Stalin (1920s): "I consider it completely unimportant who in the party will vote, or how; but what is extraordinarily important is this who will count the votes, and how."

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Here comes 8 different winner selection methods for ranked ballots.

Plurality

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Whoever gets the most first place votes is the winner.

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All rankings except first place are ignored.

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Use this in a "lesser of evils" election.

Borda count

Everybody gets points:

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for n candidates:

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- a first place vote is worth n points

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- ...
- a last place vote is worth 1 point

Borda count

So if the candidates are A, B, C and the votes are like this:

1	3	2	4
A	B	C	A
B	C	A	C
C	A	B	B

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A gets: $1 \times 3 + 3 \times 1 + 2 \times 2 + 4 \times 3 = 22$ points

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B gets: $1 \times 2 + 3 \times 3 + 2 \times 1 + 4 \times 1 = 17$ points

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C wins.

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Do several rounds.

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B	C	A	C
C	A	B	B

In the first round, we eliminate C.

Eliminating C looks like:

1	3	2	4
A	B	C	A
B	C	A	C
C	A	B	B

$\rightarrow$

1	3	2	4
A	B	A	A
B	A	B	B

Eliminating C looks like:

$$\begin{array}{cccc} 1 & 3 & 2 & 4 \\ \hline A & B & C & A \\ B & C & A & C \\ C & A & B & B \end{array} \rightarrow \begin{array}{cccc} 1 & 3 & 2 & 4 \\ \hline A & B & A & A \\ B & A & B & B \end{array} = \begin{array}{cc} 7 & 3 \\ \hline A & B \\ B & A \end{array}$$

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This method is used in Australia, Ireland, and a few local elections in US.

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Are these all equivalent? no

Pairwise comparisons

Pit the candidates against each other one-on-one in all possible matchups

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Whoever wins the most of these wins the election.

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But a person with $x\%$ support will win the election with probability $x\%$, which doesn't sound too bad.

A little digression

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Voting was not viewed as an important component of democracy.

A true government "of the people" should be made up of ordinary people, chosen at random.

Results!

Let's see the results of our election.

Results!

Let's see the results of our election.

Moral of the story:

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Moral of the story:

Different reasonable voting methods produce different outcomes.

Fairness

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Hopefully we can come up with some basic principles for fairness, and choose a system which satisfies them all.

I've got 3 basic categories for fairness:

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Let's talk some specific ways to measure these kinds of fairness.

Preferences-based fairness

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These are an attempt to define specifically the idea that the winner should be preferred over the losers

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Not satisfied by Borda count:

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In the Borda count, A gets 15 and B gets 19.

Here, A is ranked first by a majority, but B wins in the Borda count.

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A candidate like this would be preferred by a majority *when compared individually to anybody else*.

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This is also a very reasonable fairness criterion.

Twiddle-Dee & Twiddle-Dum

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Others	40,579

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Nader is typically described as "far left" on most issues, and it's fair to say most of his voters would have preferred Gore over Bush.

So if there had been preferences recorded at the ballot, they might've looked like this:

2,912,790	2,912,253	97,488
B	G	N
G	B	G
N	N	B

2,912,790	2,912,253	97,488
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2,912,790	2,912,253	97,488
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N	N	B

Here, Gore is a Condorcet winner.

2,912,790	2,912,253	97,488
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Here, Gore is a Condorcet winner.

But Bush is the plurality winner.

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Decisions-based fairness

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We'll formalize the idea that if someone switches their vote, the election outcome should change "appropriately"

Monotonicity

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This is satisfied by plurality and Borda count, so they seem pretty fair.

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Say I rank them: Obama, Romney, Johnson, Stein.

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Imagine this election between Romney & Obama, with some third parties:

Say I rank them: Obama, Romney, Johnson, Stein.

Say Romney wins, then I say "wait! I meant Obama, Romney, Stein, Johnson!"

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Say I rank them: Obama, Romney, Johnson, Stein.

Say Romney wins, then I say "wait! I meant Obama, Romney, Stein, Johnson!"

This is an "irrelvant alternative".

Irrelevant Alternatives

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In a fair system, this kind of change should not affect the election results.

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2,912,790	2,912,253	97,488
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Bush is the plurality winner.

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But the plurality system does not satisfy this.

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So the plurality system does not satisfy the irrelevant alternatives criterion.

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One more fairness criterion, of the "Honesty" type.

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If your system is not strategy-proof, the voters need to think carefully about voting "tactically", rather than voting their true preferences.

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The Nader voters would have a better outcome if they'd voted for Gore.

Their honesty caused Bush to win, which was their last choice.

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This makes politicians always claim that they’re winning.

This makes the two parties indestructible.

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This is true in our world with very few exceptions. (Canada, UK)

Criteria summary

This can all be worked out:

	Maj.	Cond.	Mono.	IA	Strategy-proof
Plurality/ Anti-plurality	✓	×	✓	×	×
Borda	×	×	✓	×	×
Instant runoff / Coombs	✓	×	×	×	×
Baldwin	✓	✓	×	×	×
Pairwise Comparison	✓	✓	✓	×	×
Random dictator	×	×	✓	✓	✓

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And one can discuss the degree of failure on various criteria.

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There are two classic "impossibility theorems" which show that no system can obey all of these.

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Arrow (1950s): No voting system can satisfy the Condorcet criterion and the irrelevant alternatives criterion.

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When choosing a voting system, we have to decide whether we want Condorcet or IA. You can't have both. (Plurality has neither.)

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We want a voting system such that:

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All the votes are symmetric- let's imagine that A is chosen as the winner.

1	1	1
A	B	C
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A wins.

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Since our system obeys the irrelevant alternatives criterion, A will still win in:

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Since our system obeys the irrelevant alternatives criterion, A will still win in:

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But now C is a Condorcet winner, so C must win because our system obeys the Condorcet criterion.

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But we just said A wins, so this is a contradiction.

The Gibbard-Satterthwaite theorem

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The first two are obviously unreasonable for real voting systems, so the summary is:

No reasonable voting system is strategy-proof.

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They are mathematically unavoidable.

So what should we do?

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We shouldn't abandon voting.

Should we continue to use the plurality system?

Pros:

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It's not. It's caused by our use of the plurality system.

Will Democratic and Republican politicians ever seriously consider dismantling the plurality system?

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The system which voters don't even think about, but the parties depend on for survival?

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Picture from User:Durova at Wikimedia Commons, CC-BY-SA

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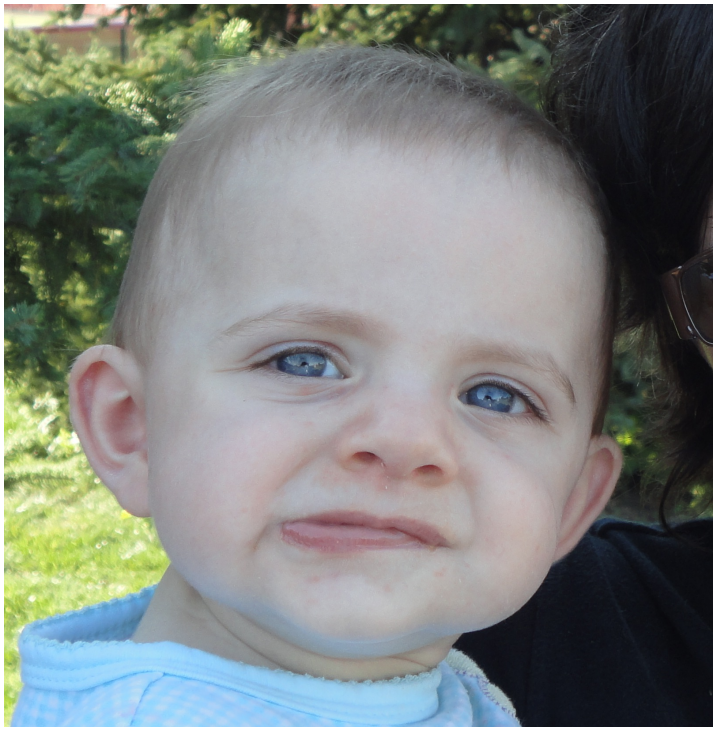

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Picture from Joel Telling at Flickr, CC-BY-SA

The end!

Read Wikipedia "Voting system" for lots more info and references.

`http://faculty.fairfield.edu/cstaecker` for these slides